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# Principal Component Analysis

Suppose we have a $m \times n$ matrix $A$ containing neural spike data, where the $i$th row vector $\mathbf{a}_i^T$ corresponds to the trace from the $i$th neuron. We define the zero-mean matrix $X$ for the data as

$$X = \begin{bmatrix} \mathbf{x}_1^T \\ \vdots \\ \mathbf{x}_m^T \end{bmatrix} = \begin{bmatrix} \mathbf{a}_1^T - \bar{\mathbf{a}}^T \\ \vdots \\ \mathbf{a}_m^T - \bar{\mathbf{a}}^T \end{bmatrix}, \bar{\mathbf{a}}^T = \frac{1}{m}\sum_{i=1}^{m} \mathbf{a}_i^T$$

Note that looking downward through each column gives us zero-mean variables.

Recall from last lecture that there exists a transformation for $X$, $Y = XQ$, such that $Y^T Y$ is diagonal. This is equivalent to decorrelating the variables in our data. Last time, we also found that the transformation $Q$ was actually $V$, the right singular vector (RSV) matrix in the singular decomposition of $X$: $X = U\Sigma V^T$.

Proof that $Y^T Y$ is diagonal:

$$\begin{aligned} Y &= XV \\ Y^T Y &= V^T X^T X V \\ &= V^T (V\Sigma^T U^T U \Sigma V^T) V \\ &= V^T (V \Sigma^T \Sigma V^T) V \\ &= V^T V \Lambda V^T V \\ &= \Lambda \end{aligned}$$

where $\Lambda$ is the eigenvalue matrix of $X^T X$ and $X^T X$ is the covariance matrix associated with $X$.

To separate out the spikes from the noise, we perform principal component analysis (PCA) on the zero-mean data:

$$
\begin{aligned}
Y &= XV \\
&= \begin{bmatrix} \mathbf{x}_1^T \\ \vdots \\ \mathbf{x}_m^T \end{bmatrix} \begin{bmatrix} \mathbf{v}_1 & \cdots & \mathbf{v}_n \end{bmatrix} \\
\begin{bmatrix} \mathbf{y}_1 & \cdots & \mathbf{y}_n \end{bmatrix} &= \begin{bmatrix} \mathbf{x}_1^T\mathbf{v}_1 & \cdots & \mathbf{x}_1^T\mathbf{v}_n \\ \vdots & \ddots & \vdots \\ \mathbf{x}_m^T\mathbf{v}_1 & \cdots & \mathbf{x}_m^T\mathbf{v}_n \end{bmatrix} \\
&= \begin{bmatrix} \langle \mathbf{x}_1, \mathbf{v}_1 \rangle & \cdots & \langle \mathbf{x}_1, \mathbf{v}_n \rangle \\ \vdots & \ddots & \vdots \\ \langle \mathbf{x}_m, \mathbf{v}_1 \rangle & \cdots & \langle \mathbf{x}_m, \mathbf{v}_n \rangle \end{bmatrix}
\end{aligned}
$$

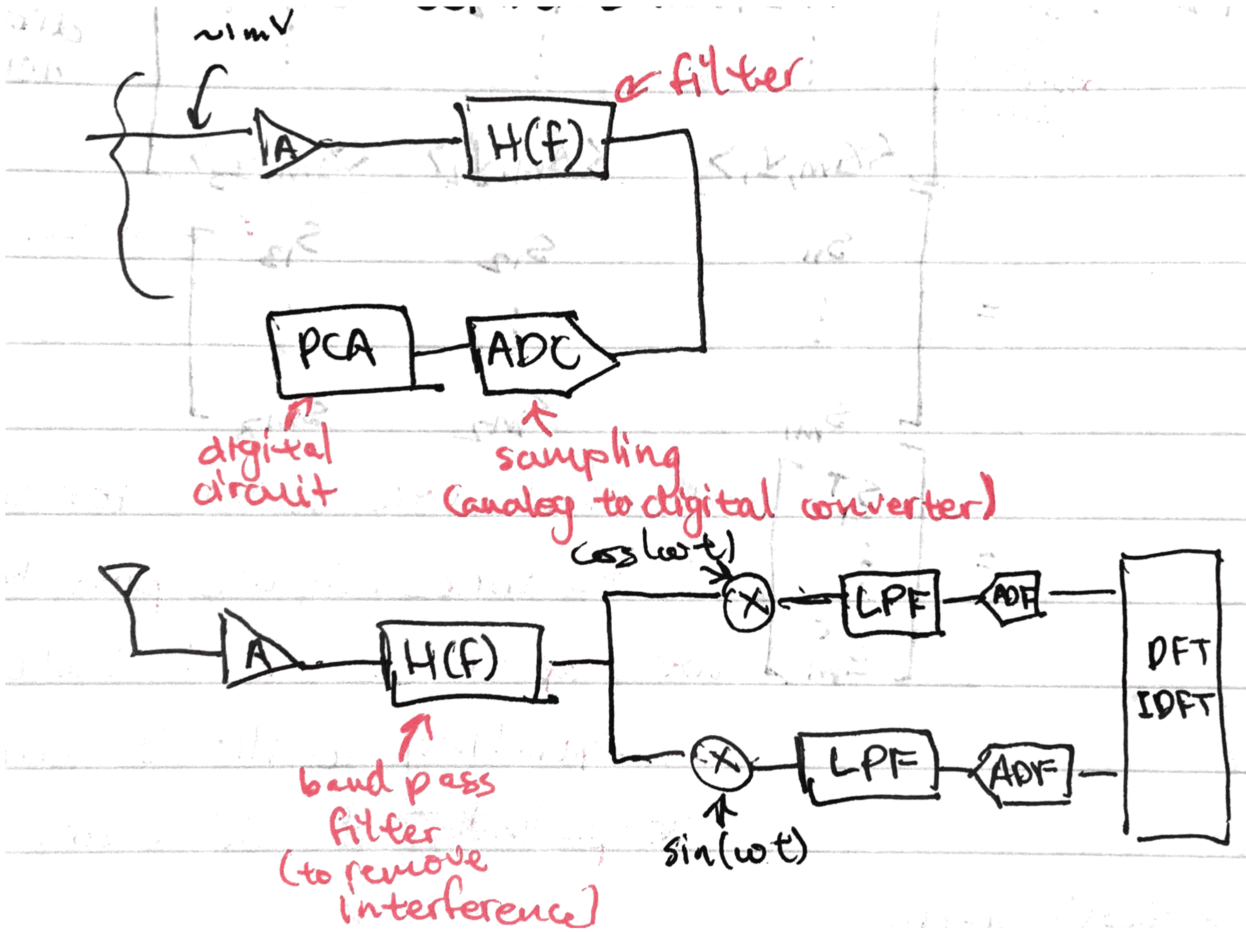
We can see that the columns of $V$ are the important components of the data. By definition, the column vectors of $V$ are arranged in decreasing order, and the first 2 or 3 $\mathbf{v}$ vectors are usually significant. Focusing on the first 3 columns of $Y$, we have

$$
\begin{aligned}
\hat{Y} &= \begin{bmatrix} \mathbf{y}_1 & \mathbf{y}_2 & \mathbf{y}_3 \end{bmatrix} \\
&= \begin{bmatrix} \langle \mathbf{x}_1, \mathbf{v}_1 \rangle & \langle \mathbf{x}_1, \mathbf{v}_2 \rangle & \langle \mathbf{x}_1, \mathbf{v}_3 \rangle \\ \vdots & \ddots & \vdots \\ \langle \mathbf{x}_m, \mathbf{v}_1 \rangle & \langle \mathbf{x}_m, \mathbf{v}_2 \rangle & \langle \mathbf{x}_m, \mathbf{v}_3 \rangle \end{bmatrix} \\
&= \begin{bmatrix} s_{11} & s_{12} & s_{13} \\ \vdots & \vdots & \vdots \\ s_{m1} & s_{m2} & s_{m3} \end{bmatrix} \\
&= \begin{bmatrix} \mathbf{s}_1^t \\ \vdots \\ \mathbf{s}_m^T \end{bmatrix}
\end{aligned}
$$

where the $\mathbf{s}$ vectors can be thought of as scores for each of the traces. If we plot these 3D coordinates, we would expect to see 3 clusters corresponding to the 3 neurons we have determined are important to our data.

## Electrical Implementation

A rough block diagram of how we would implement PCA in an electrical circuit.


~1mV
A
H(f)
filter
PCA
ADC
digital circuit
sampling (analog to digital converter)
cos(ωt)
LPF
ADF
DFT
IDFT
sin(ωt)
band pass filter (to remove interference)